Control

20/10/2015 الثلاثاء د. عرفة محاضرة [3]

Ex: $GH(s) = \dfrac{K}{s(s^2+4s+13)}$

① Poles $\Rightarrow 0, -2 \pm j3$

Zero $\Rightarrow \phi$

② s-plane

③ Real part

$0 \longrightarrow -\infty$

④ Asymptotes

①' no. of asy. $= 3 - 0 = 3$

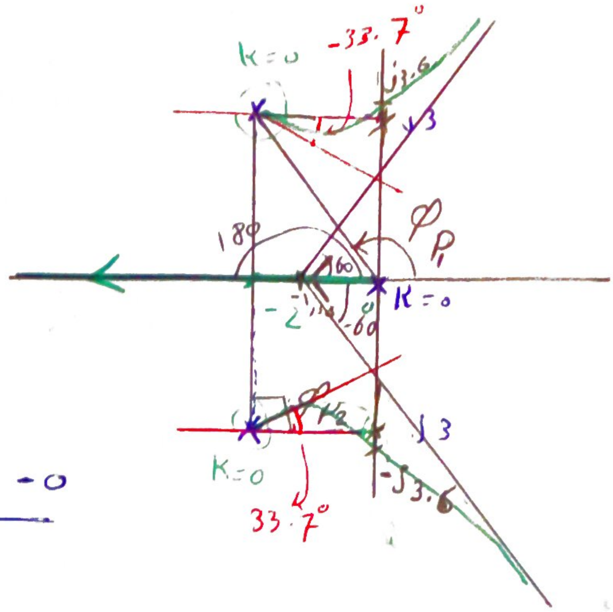
② $\sigma_c = \dfrac{0 - 2 \cancel{+j3} - 2 \cancel{-j3} - 0}{3}$

$= \dfrac{-4}{3} = -1.33$

$\theta = \dfrac{(2L+1)\,180}{3}$

$\theta_1 = 60, \quad \theta_2 = 180, \quad \theta_3 = 300 = -60^\circ$

خطوة ال Breaking point غير ضرورية حيث أنه لا توجد



⑤ departure angle

$\theta_D = 180 - \phi_p + \phi_z$

departure ← D

$\theta_{D_{-2+j3}} = 180 - (\phi_{P_1} + \phi_{P_2}) + 0$

$= 180 - \left[(180 - \tan^{-1} 3/2) + 90^\circ\right] + 0$

$= -33.7^\circ$

$\theta_{D_{-2-j3}} = 33.7^\circ$

⑥ Range of stability for k

At img Axis

ch.eq. $\Rightarrow 1 + GH(s) = 0$

$$1 + \frac{K}{s^3 + 4s^2 + 13s} = 0$$

$$s^3 + 4s^2 + 13s + K = 0$$

s^3	1	13
s^2	4	K
s^1	$\frac{52-K}{4} > 0$ ①	
s^0	$K > 0$ ②	

for ①

$52 - K > 0$

$K < 52$

range of stability

$0 < K < 52 \rightarrow K_c$

Aux eq $A(s)$

$A(s) = 4s^2 + K$

$4s^2 + K_c = 0$

$4s^2 + 52 = 0 \Rightarrow s_{1,2} = \pm j\,3.6$

تقاطع ال root locus مع المحور الرأسي

Ex: $GH(s) = \frac{K(s+2)}{s^2 + 4s + 20}$

① Poles $\Rightarrow -2 + j4,\ -2 - j4$

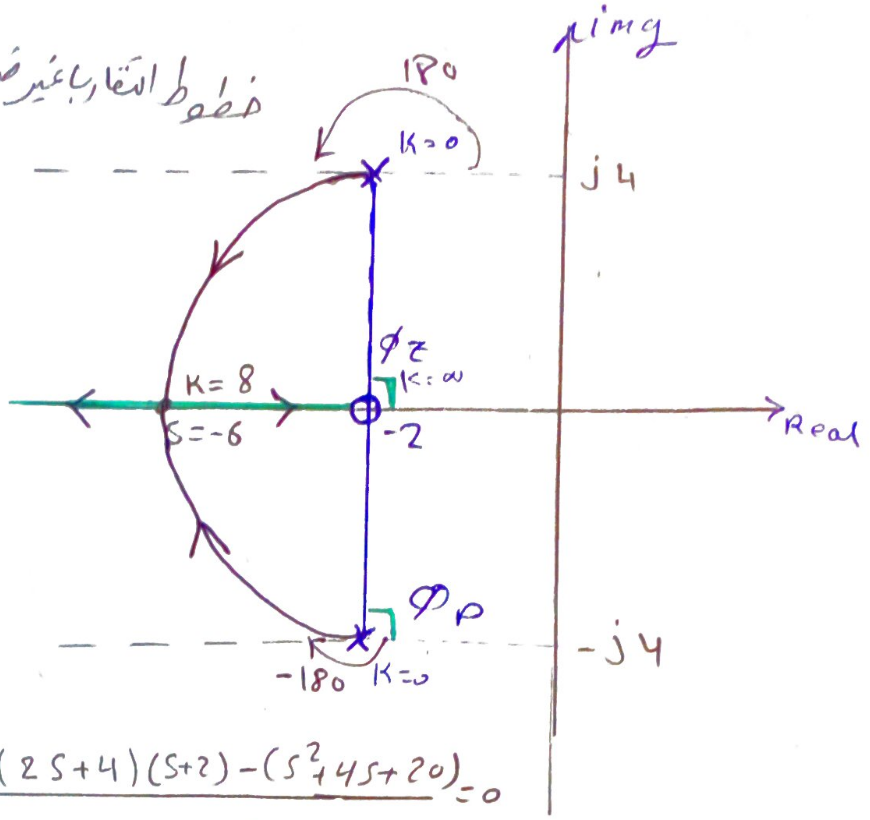
② Zeros $\Rightarrow -2$

② s-plane $\Rightarrow$ turn over

③ Real Part

$-2 \longrightarrow -\infty$

خطوط التقارب غیر ضروریة



④ Breaking Point

Ch. eq. $1+GH(s)=0$

$GH(s) = -1$

$$K = -\frac{s^2+4s+20}{s+2}$$

$$\frac{dK}{ds} = 0 \Rightarrow -\frac{(2s+4)(s+2)-(s^2+4s+20)}{(s+2)^2} = 0$$

$$\therefore (s+2)(2s+4) - (s^2+4s+20) = 0$$

$$2s^2+8s+8-s^2-4s-20=0$$

$$s^2+4s-12=0$$

$$(s+6)(s-2) \Rightarrow s_{1,2} = \overset{\times}{2}, \; -6$$

Accepted

Breaking point (s_b)

$$s_b = -6 \Rightarrow K\Big|_{s_b=-6} = \frac{(-6)^2-4(6)+20}{-6+2} = 8$$

⑤ departure angle

$$\theta_D = 180 - \phi_p + \phi_z$$

$$\phi_{D_{-2+j4}} = 180 - 90 + 90 = 180^\circ$$

$$\phi_{D_{-2-j4}} = -180^\circ$$

Stable for $K>0$

Ex: $GH(s) = \dfrac{20(1+Ks)}{s(s+1)(s+4)}$

Ch. eq:

$1 + GH(s) = 0$

$1 + \dfrac{20 + 20Ks}{s(s+1)(s+4}$

$\boxed{s(s+1)(s+4) + 20} + 20Ks = 0$

$1 + \underbrace{\dfrac{20Ks}{s(s+1)(s+4)+20}}_{GH(s)} = 0$

$GH(s) = \dfrac{20Ks}{s(s+1)(s+4)+20}$

Poles $\Rightarrow -5, j2, -j2$

Zero $\Rightarrow 0$

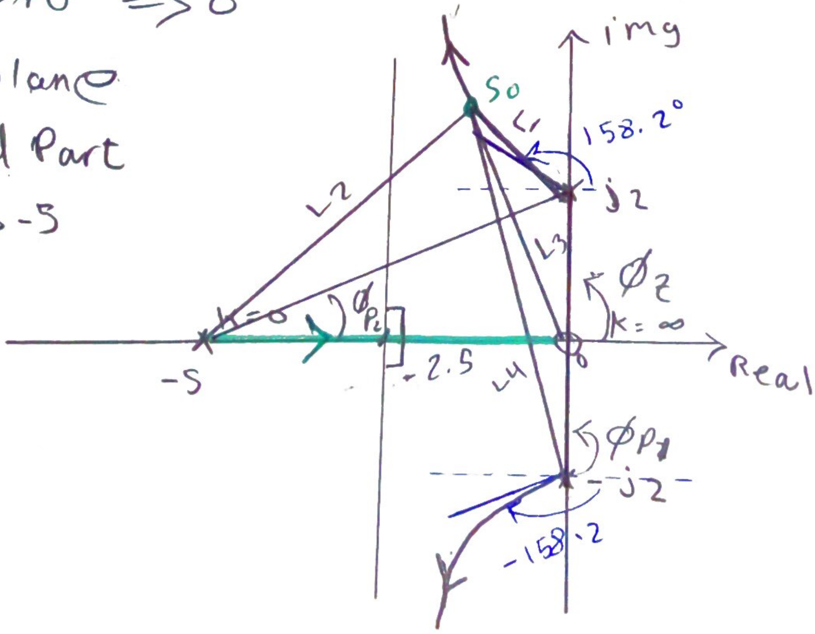
① Poles $\Rightarrow -5,\ j2,\ -j2$

Zero $\Rightarrow 0$

② s-plane

③ Real Part

$0 \rightarrow -5$



$\Rightarrow$ Turn over

④ Asy. → ① No. of asy. = 3 − 1 = 2

→ ② $C_A = \dfrac{(-5 + j2 - j2) - (0)}{2}$

$= \dfrac{-5}{2} = -2.5$

→ ③ $\theta = \dfrac{(2L+1)\,180}{2}$

$\theta_1 = 90$

$\theta_2 = -90$

⑤ departure Angle :

$\theta_{D_{j2}} = 180 - (\phi_{P_1} + \phi_{P_2}) + \phi_z$

$= 180^\circ - (90 + \tan^{-1}(\frac{2}{5})) + 90^\circ$

$= 158.2^\circ$

$\theta_{D_{-j2}} = -158.2^\circ$

System is stable for $K > 0$

$\boxed{1}$ Find K at S_o

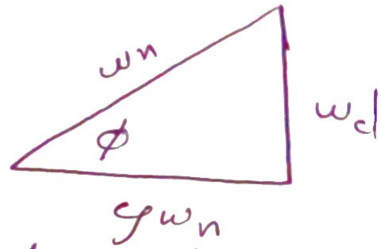
$$K\Big|_{S_o} = \frac{\prod \text{Poles}}{\prod \text{zero}} = \frac{L1 . L2 . L3}{L4}$$

$\boxed{2}$ find K at $\zeta = 0.5$

$$T.F. = \frac{\omega_n^2}{S^2 + 2\zeta\omega_n S + \omega_n^2}$$

$$S_{1,2} = -\zeta\omega_n \pm j\omega_d$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$



$\boxed{2}$ K at $\zeta = 0.5$

$$\phi = \cos^{-1} \zeta = 60^\circ$$

$\phi = \cos^{-1} \zeta$

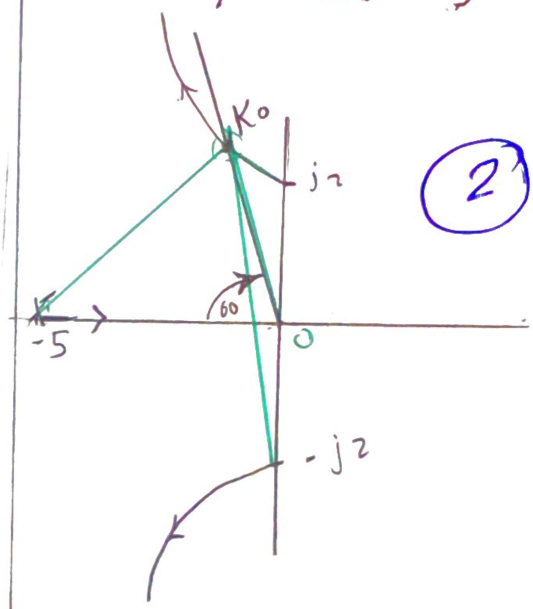
$\boxed{3}$ K at $t_s = \checkmark$ sec

given $\zeta \omega_n$

$$t_s = \frac{4}{\zeta \omega_n} \quad (2\% \text{ error})$$

ده الافتراضي لو مذكرش الخطأ المطلوب

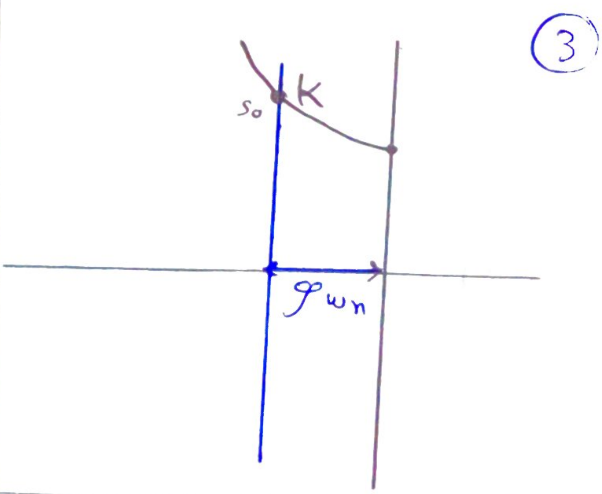
هات ال $\zeta \omega_n$

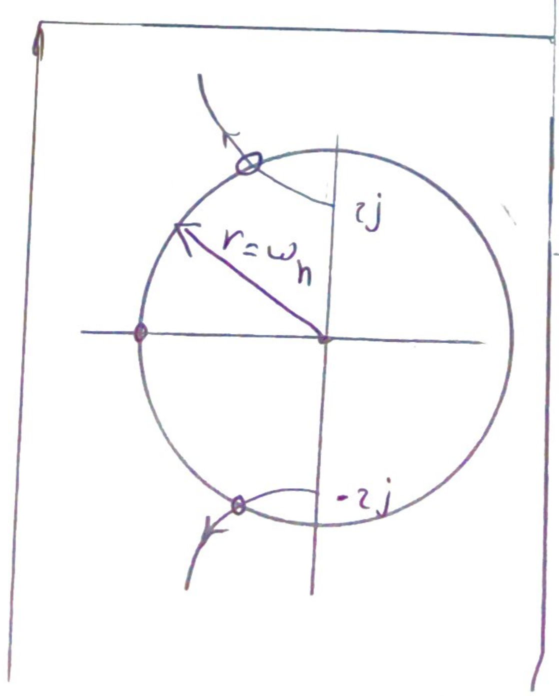


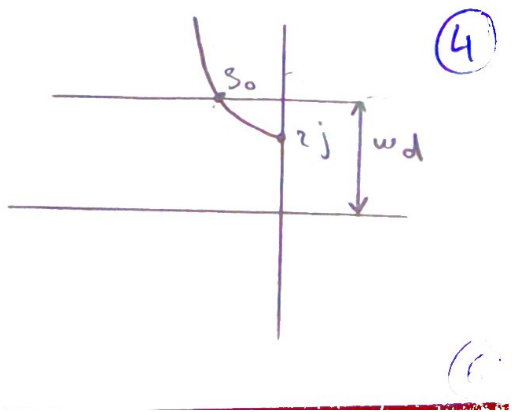
④ K at $\omega_d = \checkmark$ rad/sec

damp. freq. →

⑤ find K $\omega_n =$ ——— rad sec



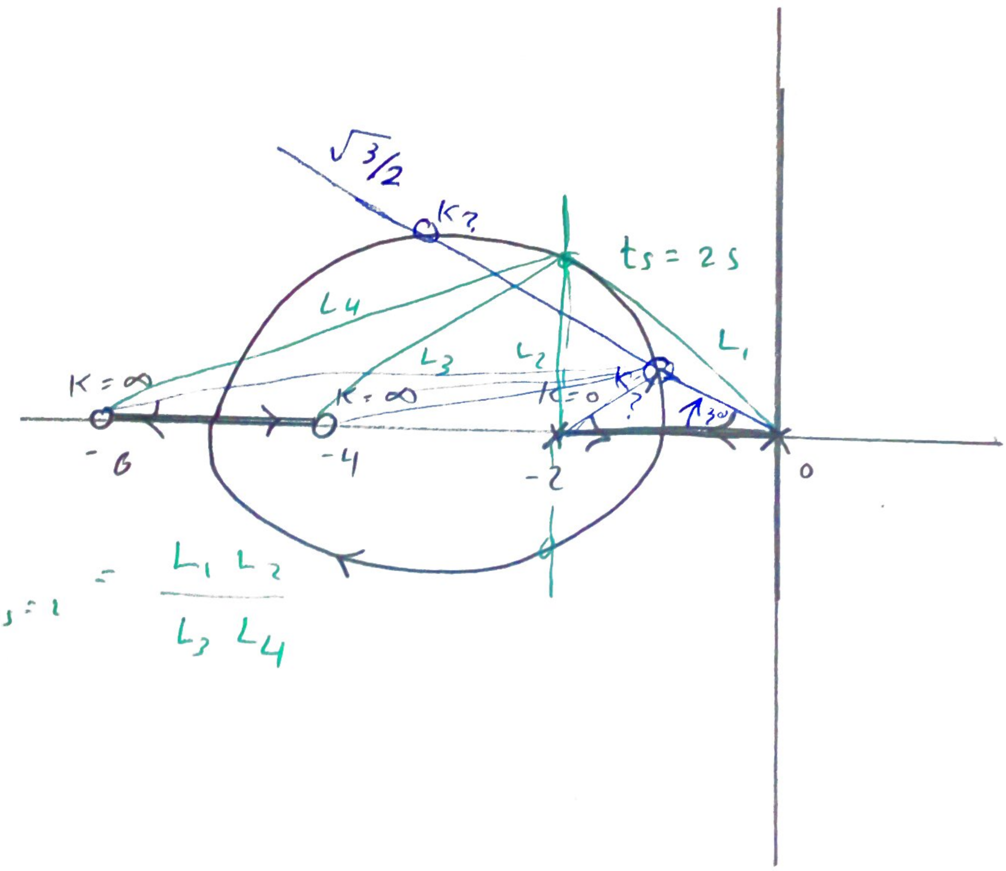




Report:

$$GH(s) = \frac{K(s+4)(s+6)}{s(s+2)}$$

- Draw the root locus
- Find the range of K for stability
- K at $t_s = 2$ sec $= \frac{4}{\zeta\omega_n} \Rightarrow \zeta\omega_n = 2$
- K at $\zeta = \frac{\sqrt{3}}{2} \Rightarrow \phi = \cos^{-1}\zeta = 30$



$$K_{t_s=2} = \frac{L_1 L_2}{L_3 L_4}$$